Correction

Addendum and Correction to "Optimal Phases for a Family of Quadriphase CDMA Sequences"

Harry Leib, *Senior Member, IEEE*, and
Steven R. Weller, *Member, IEEE*

This correspondence presents several corrections and an addendum to.[1]

Correction 1

Equation (7) should read

$$\sum_{l=0}^{L-1}|C(x,y)(l)|^2+\sum_{l=0}^{L-1}|C(x,y)(l-L)|^2 = \sum_{l=0}^{L-1}C(x,x)(l)[C(y,y)(l)]^* + \sum_{l=0}^{L-1}C(x,x)(l-L)[C(y,y)(l-L)]^*. \quad (7)$$

Correction 2

There is an error in the second equation following (19). The revised text should read as follows:

In view of (13), the above is actually equal to

$$\sum_{l=1}^{L-2}(L+1)(L-l)=(L+1)^2(L-2)/2.$$

Similarly, we can show that

$$\sum_{l=1}^{L-2}\sum_{x\in U_\alpha}|C(x,x)(l+1)|^2=(L^2-1)(L-2)/2.$$

Thus the right-hand side of (19) is equal to $L(L+1)(L-2)$. Substituting these results into (17), we have the average user interference

$$\frac{1}{A}\frac{\binom{L-1}{A-2}}{\binom{L+1}{A}}\sum_{y\in U_\alpha}\sum_{x\in U_\alpha;\, x\neq y}(6L^3)^{-1}[2\mu_{x,y}(0)+\mathrm{Re}\{\mu_{x,y}(1)\}] \le \frac{A-1}{3L}\left(1-\frac{1}{2L}\right) \quad (20)$$

and

$$\frac{1}{A}\frac{\binom{L-1}{A-2}}{\binom{L+1}{A}}\sum_{y\in U_\alpha}\sum_{x\in U_\alpha;\, x\neq y}(6L^3)^{-1}[2\mu_{x,y}(0)+\mathrm{Re}\{\mu_{x,y}(1)\}] \ge \frac{A-1}{3L}\left(1-\frac{3L-4}{2L^2}\right) \quad (21)$$

when there are A active users out of $L+1$ possible users. The difference between the upper and lower bounds is only $(A-1)(L-2)/3L^3$.

Substituting the upper and lower bounds of (20) and (21) into (18) leads, respectively, to the lower bound on the average signal-to-noise ratio

$$\left\{\frac{A-1}{3L}\left(1-\frac{1}{2L}\right)+N_0/2E_b\right\}^{-1/2} \quad (22)$$

and the upper bound

$$\left\{\frac{A-1}{3L}\left(1-\frac{3L-4}{2L^2}\right)+N_0/2E_b\right\}^{-1/2}. \quad (23)$$

Correction 3

The expression for the average user interference with ideal random sequences from [22] that is used in the above paper[1] after (23) is incorrect [1]. The correct expression is the one from [13] in the original paper

$$\frac{A-1}{3L}$$

which in fact improves the results from the above paper.[1]

Correction 4

As a consequence of the corrected bounds (22), (23), the values of several numerical quantities in Section VII need revising. Equation (42) and its successor should read

$$\{0.04422\cdot(A-1)+N_0/2E_b\}^{-1/2}$$

and

$$\{0.03936\cdot(A-1)+N_0/2E_b\}^{-1/2}. \quad (42)$$

Likewise, the text appearing immediately under Fig. 5 should read:

whereas the lower and upper bounds of (22) and (23) are, respectively,

$$\{0.02148\cdot(A-1)+N_0/2E_b\}^{-1/2}$$

and

$$\{0.02020\cdot(A-1)+N_0/2E_b\}^{-1/2}.$$

Correction 5

In view of the correction to the expression for the average user interference with ideal random sequences, the corresponding numerical result from (43) should read

$$\{0.047619\cdot(A-1)+N_0/2E_b\}^{-1/2} \quad (43)$$

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H. Leib is with the Department of Electrical Engineering, McGill University, Montréal, P.Q., Canada, H3A 2A7.
S. R. Weller is with the Department of Electrical and Computer Engineering, University of Newcastle, Callaghan, NSW 2308, Australia.
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[1] F.-W. Sun and H. Leib, "Optimal phases for a family of quadriphase CDMA sequences," *IEEE Trans. Inform. Theory*, vol. 43, pp. 1205–1217, July 1997.

and the second equation after (44) should read

$$\frac{1}{\sqrt{0.047619(A-1)}}.$$

The fifth equation after (44) that gives the largest achievable gain of the sequences from Table I with respect to random sequences should read

$$10 \log (0.047619/0.04123) = 0.63 \text{ dB}$$

whereas the subsequent equation that gives the loss of the sequences from Table II with respect to random sequences should read

$$10 \log (0.055185/0.047619) = 0.64 \text{ dB}.$$

Addendum

The scope of this addendum is to clarify some issues related to Section V of the above paper.[1] Let U be a cardinality A subset of U_α, the set of sequences considered in Section V. The expected value of the average user interference of the subset U is

$$r_U = \frac{1}{A} \sum_{y \in U} \sum_{x \in U; x \neq y} (6L^3)^{-1} [2\mu_{x,y}(0) + \mathrm{Re}\{\mu_{x,y}(1)\}].$$

The equation before (17) in the above paper[1] further averages r_U also over all subsets U of U_α. This average, denoted by $\overline{r}_U$, is equal to (17). In the absence of an explicit expression for (17), the above paper[1] presents upper and lower bounds (20) and (21) to $\overline{r}_U$. Therefore, the set U_α contains at least one subset U with r_U not larger than (20) that is less than $(A-1)/(3L)$ the average user interference for random sequences. A similar result for Gold binary sequences is known [2].

If we consider now A users employing the sequences from subset U, then r_U gives an indication to the multiuser interference that a typical user experiences. The interference of the most favored user is less than r_U, while the interference of the least favored user is more than r_U. Optimization of the sequence phases may result in a further reduction of this interference.

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References

[1] D. V. Sarwate, “Comments on ‘An alternative derivation for the signal-to-noise ratio of a SSMA system’,” *IEEE Trans. Commun.*, vol. 43, p. 2903, Dec. 1995.

[2] ——, “Mean-square correlation of shift-register sequences,” *Proc. Inst. Elec. Eng.*, pt. F, vol. 131, no. 2, pp. 101–106, Apr. 1984.